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Fourth Solution.

From A draw at random the chord AE', put $\angle A'AE'=\Psi$; then $AD'=2a\cos\Psi=r_1$, and $AE'=4a\cos\Psi=r_2$. For any value of Ψ the number of *favorable* chances is represented by the *sectoral* surface PAD', and the total number of chances by the sectoral surface $A'AE'$. The chance in consideration, therefore, becomes

$$C=2\int_0^{\frac{1}{2}\pi}\int_0^{r_1}d\Psi rdr\div2\int_0^{\frac{1}{2}\pi}\int_0^{r_2}d\Psi rdr=\frac{1}{4}.=.25.$$

Fifth Solution.

Put $\angle PAD'=\omega$, then $\angle APD'=(\frac{1}{2}\pi-\omega)$. Therefore, for any range PD' the projectiles falling on the circular arc DMD' are within the field. Consequently the required chance becomes

$$C=4a\int_0^{\frac{1}{2}\pi}(\tfrac{1}{2}\pi-\omega)\sin\omega d\omega\div4\pi a\int_0^{\frac{1}{2}\pi}\sin\omega d\omega=\frac{1}{2}-\frac{1}{\pi},=.182.$$

Sixth Solution.

The number of *favorable* chances is proportional to $2\angle MPD'=2(\frac{1}{2}\pi-\Psi)$, and the total number of chances is proportional to $2(\pi)$. Hence the required chance becomes

$$C=2\int_0^{\frac{1}{2}\pi}(\tfrac{1}{2}\pi-\Psi)d\Psi\div2\pi\int_0^{\frac{1}{2}\pi}d\Psi=\tfrac{1}{4},=.25.$$

NOTE—Since the projectiles are *thrown* at random, they should *fall* at random; and, therefore, the required chance should be $C=\frac{1}{4},=.25$. To interpret *this* result is apparently easy enough; but to interpret Todhunter's result, or the results $C=.297+$and $C=.182$, is not so easy. In fact, the interpretation of these three results becomes all the more remarkable when we note that their average value $C_A=.238+$, which average value differs but slightly from Todhunter's result.

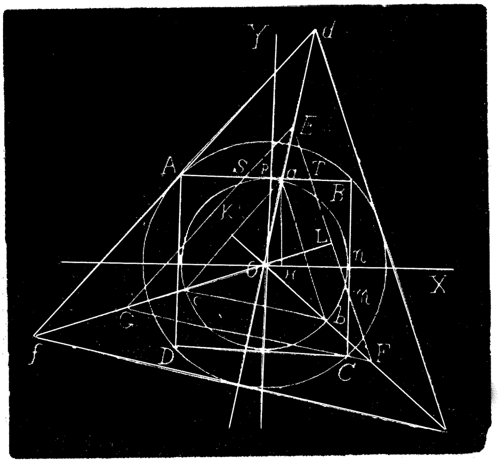
This problem was also solved by G. B. M. Zerr. J. M. Colaw, and John Dolman, Jr., their result agreeing with that given by Todhunter and the first solution of Professor Matz. Professor Zerr says this result is perfectly true as to mathematical fact. We published all of Professor Matz's solutions for comparison.—EDITOR.

18. Proposed by B. F. FINKEL, A. M., Professor of Mathematics in Kidder Institute, Kidder, Missouri.

What is the average volume common to a cube and a rectangular solid one inch square, the axis of rectangular solid being equal to and coinciding with the diagonal of the cube?

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $ABCD$ be a section of the rectangular solid, EFG a section of



the cube both projected upon the same plane, perpendicular to the diagonal of the cube. The triangle varies in size from *abc*, the inscribed triangle of the inscribed circle of the square, to *def* the circumscribed triangle of the circum-circle of the square. Before the triangle is equal in area to *abc*, all the cube is common to both, after the triangle is greater in area than *def*, all the rectangular solid is common to both.

Let a=edge of cube, c= side of square *ABCD*. Also suppose $c<\frac{a}{\sqrt{3}}$; for if $c>\frac{a}{\sqrt{3}}$ the section of the cube will be both a triangle and a hexagon.

Let x=side of triangle, p=perpendicular from corner of cube to triangle *EFG*. Then $p=\frac{x}{\sqrt{6}}$ when $x=ab=\frac{c\sqrt{3}}{2}$, $p=\frac{c}{2\sqrt{2}}$, area $abc=\frac{3\sqrt{3}c^2}{16}$

Volume of pyramid common to both $=\frac{1}{3}p\times abc=\frac{c^3\sqrt{3}}{32\sqrt{2}}$.

When $x=de=c\sqrt{6}$, $p=c$.

$\therefore$ volume rectangular solid common to both$=c^2(\frac{1}{2}a\sqrt{3}-c)=\frac{1}{2}c^2(a\sqrt{3}-2c)$.

$\therefore$ the constant volume common to both solids is double the two volumes just found as we have considered but half the cube.

$$\therefore\quad V_1=2\left\{\frac{c^3\sqrt{3}}{32\sqrt{2}}+\tfrac{1}{2}c^2(a\sqrt{3}-2c)\right\}=\frac{c^2}{16\sqrt{2}}\left\{c\sqrt{3}+16a\sqrt{6}-32c\sqrt{2}\right\}.$$

To find the average volume common to both, that varies, let *OX*, *OY* be the axis of reference, $\angle EOY=\theta$.

This volume has sections ranging from the triangle through the quadrilateral, pentagon, hexagon, heptagon, and back through its variations to the square. To find this volume I shall divide its altitude into three equal parts and find the average area of sections passing through these points of division, and then apply the formula of approximate cubature.

For the first point of division $x=\frac{c\sqrt{3}}{3}(\sqrt{2}+1)$.

For the second point of division $x=\frac{c\sqrt{3}}{6}(4\sqrt{2}+1)$.

$OL=OK=\frac{x\sqrt{3}}{6}$, $EH=\frac{x\sqrt{3}}{3}\cos\theta$.

Equation to EF is $x_1\cos\left(\frac{\pi}{6}-\theta\right)+y_1\sin\left(\frac{\pi}{6}-\theta\right)=\frac{x\sqrt{3}}{6}$.

Equation to EG is $x_1\cos\left(\frac{5\pi}{6}-\theta\right)+y_1\sin\left(\frac{5\pi}{6}-\theta\right)=\frac{x\sqrt{3}}{6}$.

$$RT=\frac{2x\sqrt{3}-3c\cos\theta+3c\sqrt{(3)}\sin\theta}{6[\sqrt{(3)}\cos\theta+\sin\theta]},\ RS=\frac{2x\sqrt{3}-3c\cos\theta-3c\sqrt{(3)}\sin\theta}{6[\sin\theta-\sqrt{(3)}\cos\theta]}.$$

$$nm=\frac{2x\sqrt{3}-3c\sqrt{(3)}\cos\theta-3c\sin\theta}{6[\cos\theta-\sqrt{(3)}\sin\theta]}.$$

When $x=\frac{c\sqrt{3}}{3}(\sqrt{2}+1)$ the triangle EFG is greater than the inscribed triangle of circum-circle of the square $ABCD$. Hence three times the average area of EST subtracted from the area EFG gives the average area of the section required. Call this area A_2. From $\theta=0$ to $\theta=\theta_1$, area $EST=\frac{1}{2}(RT-RS)(EH-\frac{1}{2}c)$; from $\theta=\theta_1$ to $\theta=\frac{\pi}{4}$, the area $=EST+TBM$ $=\frac{1}{2}(RT-RS)(EH-\frac{1}{2}c)+\frac{1}{2}(RT-\frac{1}{2}c)(\frac{1}{2}c-nm)$. θ_1 is determined from the equation $\frac{2x\sqrt{3}-3c\cos\theta+3c\sqrt{(3)}\sin\theta}{6[\sin\theta+\sqrt{(3)}\cos\theta]}=\frac{1}{2}c$.

When $x=\frac{c\sqrt{3}}{6}(4\sqrt{2}+1)$, the triangle EFG is greater than the circumscribed triangle of the in-circle of the square $ABCD$. Hence, three times the average area of the triangle TBM subtracted from the area of the square $ABCD$ gives the average area of the section required. Call this area A_3.

Area $TBM=\frac{1}{2}(\frac{1}{2}c-nm)(\frac{1}{2}c-RT)$. Limits of θ are, $\theta=\theta_2$ to $\theta=\frac{\pi}{4}$, where θ_2 is found from the equation, $\frac{2x\sqrt{3}-3c\sqrt{(3)}\cos\theta-3c\sin\theta}{6[\cos\theta-\sqrt{(3)}\sin\theta]}=\frac{1}{2}c$.

$$\therefore\quad V_2=2\times\frac{h}{8}(A_1+3A_2+3A_3+A_4),$$

where $h=c-\frac{c}{2\sqrt{2}}=\frac{c}{2\sqrt{2}}(2\sqrt{2}-1)$. $A_1=\text{area } abc=\frac{3\sqrt{(3)}c^2}{16}$, $A_4=\text{area } ABCD=c^2$. $\therefore\ V_2=\frac{3c}{8\sqrt{2}}(2\sqrt{2}-1)(A_2+A_3)+\frac{c}{8\sqrt{2}}(2\sqrt{2}-1)\left(\frac{3\sqrt{(3)}c^2}{16}+c^2\right)=\frac{3c}{8\sqrt{2}}(2\sqrt{2}-1)(A_2+A_3)+\frac{c^3}{128\sqrt{2}}(6\sqrt{6}+32\sqrt{2}-3\sqrt{3}-16)$.

$$V=V_1+V_2=\frac{3c}{8\sqrt{2}}(2\sqrt{2}-1)(A_2+A_3)+\frac{c^3}{128\sqrt{2}}$$

$(6\sqrt{6}+5\sqrt{3}-224\sqrt{2}-16)+ac^2\sqrt{3}$, where A_2 and A_3 are found as indicated above.

[NOTE.—No other solution of this problem was received. Professor Zerr worked on this problem during the hot days of last August. He said the temperature was too high for him to make a complete solution. The problem is more difficult than we supposed. After trying to effect a solution the true character of the problem was revealed us. EDITOR.]